ECExxx Model Predictive Control Winter 2022
Days/Hours/Room TBD

Calendar Description

This course will tackle the problem of optimal control of dynamical systems with constraints. This is done through an optimization-based method called Model Predictive Control, or MPC. The course covers: 1) basic concepts of system theory, including state-estimation and hybrid systems, 2) convex optimization, constrained and unconstrained optimal control, 3) concepts of stability, reachability, invariant sets, 4) Model Predictive Control formulations, and associated mathematical guarantees on robustness, optimality and recursive feasibility, 5) numerical methods for MPC.

Instructor

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Course Outline

Increased system complexity and more demanding performance requirements have rendered traditional control laws inadequate whether simple PID loops are considered or robust feedback controllers are designed according to some H2/infinity criterion. Applications ranging from the process industries to the automotive and the communications sector are making increased use of Model Predictive Control (MPC) where a fixed control law is replaced by on-line optimization performed over a receding horizon. The advantage is that MPC can deal with almost any time-varying process and specifications, limited only by the availability of real-time computation power. In the last few years we have seen tremendous progress in this interdisciplinary area where fundamentals of systems theory, computation and optimization interact. For example, methods have emerged to handle hybrid systems, i.e. systems comprising both continuous and discrete components. Also, it is now possible to perform most of the computations off-line thus reducing the control law to a simple look-up table online.

The first part of the course is an overview of basic concepts of system theory and optimization, including hybrid systems and multi-parametric programming. In the second part we will show

Table 1: Course timeline, based on 24 lectures, each of 1.5 hours in duration. MPC is an abbreviation for Model Predictive Control, which is the focus of this course.

Class	General Topic	Specific Content
#01	Introduction and overview	Limitations of classical control, Optimization-based Control, Origins of MPC, applications
#02	System Theory Basics (1)	Models of dynamic systems, Analysis of Discrete-time Linear Systems
#03	System Theory Basics (2)	Analysis of Discrete-time Non-linear systems
#04	Model Uncertainty and State Estimation	Uncertainty modeling (stochastic and worst-case disturbances), Linear State Estimation
#05	Convex Optimization (1)	Convex sets, functions and optimization problems
#06	Convex Optimization (2)	Duality, Generalized Inequalities, connection to optimal control
#07	Unconstrained Linear Optimal Control (1)	Finite horizon, Receding Horizon Control problems
#08	Unconstrained Linear Optimal Control (2)	Solutions via dynamic programming, Infinite Horizon Control
#09	Constrained Finite Time Optimal Control (1)	State/input constraints, Predictive Control basics
#10	Constrained Finite Time Optimal Control (2)	Constrained Optimal control $(1, 2, \infty)$-norm, Quadratic Program Formulations
#11	Feasibility and Stability of MPC	Receding horizon MPC, Terminal Conditions, Stability guarantees
#12	Invariance	Recursive feasibility of MPC, Controlled Invariance, set representations
#13	Reachability and set invariance (1)	Reachable & Invariant sets, set computations
#14	Reachability and set invariance (2)	Reachability & Controllabillity, Robust MPC
#15	Practical issues in MPC	Reference tracking, Soft constraints, Generalizing MPC
#16	Explicit MPC (1)	Offline-online control, Multi-Parametric Programming (mpQP, mpLP)
#17	Explicit MPC (2)	Real-time MPC via explicit feedback laws, Computation tool
#18	Robust MPC (1)	Uncertainty models, bounded additive noise, Robust open-loop MPC
#19	Robust MPC (2)	MPC as a game, closed-loop MPC, Tube-MPC
#20	Hybrid MPC (1)	Hybrid Systems, Optimal Control of Hybrid Systems
#21	Hybrid MPC (2)	MPC and Explicit MPC for Hybrid Sytems
#22	Numerical methods (1)	Gradient and Newton methods, Preconditioning and convergence
#23	Numerical methods (2)	Alternating minimization, Interior point methods, Software
#24	MPC applications in autonomous systems	Recent research outcomes - guest lecturer (TBD)

how these concepts are utilized to derive MPC algorithms and to establish their properties. Based on the makeup of the class, domain specific examples will be formulated and analyzed as Model Predictive Control algorithms.

See table 1 for a detailed timeline of the course.

Grading

The course will consist of 4 homework assignments (weighted equally) and a final exam. The grading scheme is:

- Homeworks: 60%
- Final exam: 40%

Late Turn-in Policy

Homework will be due in class. Homework received by the Friday of the same week will be accepted with a 20% late penalty. Homework will not be accepted after the Friday.

Intended Learning Outcomes

By the end of this course, students will be able to:

1. Recognize control problems where Model Predictive Control (MPC) offers advantages over classical control methods (e.g., PID and pole-placement) and modern optimal control methods (e.g., LQR).

2. Formulate constrained optimal control problems (e.g., motion planning of robotic systems, control of chemical plants etc.) as Model Predictive Control optimizations, and deploy the correct solvers to obtain sequences of control signals.

3. Verify that closed-loop control with the designed MPC has guarantees on stability, optimality (or bounded sub-optimality), robust constraint satisfaction (state and input constraints) and recursive feasibility of the underlying optimization.

4. Implement MPC algorithms using the Multi-Parametric Optimization (MPT 3.0+) toolbox in MATLAB.

Recommended Background

While not required, it is recommended that students have a background in linear systems, control theory and convex optimization, e.g., have taken courses such as ECE 488, ECE/CO 602, ECE 682 (or equivalent) prior to enrolling in this course. The basics in linear systems and convex optimization will be reviewed in first few lectures. We will make use of the Multi-Parametric Toolbox (MPT3) for MATLAB which was developed by the automatic control group at ETH Zurich, and other universities. The student should be comfortable writing MATLAB code.

Recommended reading

- Stanford Engineering's course on Convex Optimization.
- Stanford Engineering's course on Introduction to Linear Dynamical Systems.

Textbook

F. Borrelli, A. Bemporad and M. Morari, "Predictive Control for Linear and Hybrid Systems", Cambridge University Press.
Note: The book (in pdf form) and other related material are available on Professor Francesco Borrelli's MPC course web page.

Acknowledgement

The course instructor would like to thank Professor Manfred Morari, whose slides are a basis for most of the lectures in this course.